

Decays $B_s \rightarrow J/\psi + \eta$ and $B_s \rightarrow J/\psi + \eta'$ in the framework of covariant quark model

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Abstract

The covariant quark model represents an appropriate theoretical framework to describe the recent results on $B_s \rightarrow J/\psi + \eta$ and $B_s \rightarrow J/\psi + \eta'$ decays from the Belle and LHCb collaborations. In this article we present the main features of the covariant quark model together with details on some of its aspects and methods, which we consider to be important. Further we apply the model specifically to the studied decay processes and give numerical results on decay widths as they follow from the model. We conclude that the model, with most of its parameters previously fixed from different processes, is able to incorporate the new experimental measurements. In particular, we found that the ratio of the branching fractions of the decays B_s into $J/\psi + \eta'$ and $J/\psi + \eta$ is equal to $R \approx 0.86$, in agreement with the data reported by Belle and LHCb collaborations.

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I. INTRODUCTION

The low energy region of quantum chromodynamics (QCD) is an interesting and challenging area of investigation nowadays. On one hand many accurate data exists in this domain, especially results from hadron spectroscopy and hadron decays, which might give a handle to understanding the quark dynamics inside hadrons. Presently the precision and the amount of these data continuously increases thanks to ongoing high-energy experiments and heavy-quark factories. On the other hand theoretical predictions encounter difficulties: since with decreasing energy the coupling constant increases, the perturbative approach (pQCD) loses its applicability and cannot provide valid results. Also lattice calculations are generally seen as not yet enough developed and precise to be considered as a well established low-energy solution of the QCD which would satisfactorily explain its numerous features. Undoubtedly however, some theoretical description of hadron dynamics is needed at least for a correct description of physics and background measured in particle detectors. One thus has to rely on a model-dependent approach.

In the wake of the recent measurement of the branching fractions of B_s meson decays into $J/\psi + \eta$ and $J/\psi + \eta'$ made by Belle [1] and LHCb [2] collaborations, we perform the analysis of the above processes in the framework of the covariant quark model.

Theoretical facets of these decays in the context of exploring CP violation have been discussed in the papers Refs. [3–6]. Two points are important in analyzing: $SU_f(3)$ breaking and $\eta - \eta'$ mixing. There are attempts to include in addition the admixture of glueball in the $\eta - \eta'$ mixing, see i.e. [7].

The covariant quark model [8] is an ambitious model with many appealing features. Its Lagrangian-based formulation leads to a full Lorentz invariance and, in the limit of large number of hadrons, it has only one free parameter per hadron. A distinctive feature of this approach is that the multiquark states, such as baryons (three quarks), tetraquarks (four quarks), etc., can be considered and described as rigorously as the simplest quark-antiquark systems (mesons). The model has wide application spectra and was already successfully used to describe different types of heavy hadron decays and other hadron-related observables. The last applications have been done for studying the properties of the B_s meson [9], the light baryons [10], the heavy Λ_b baryon [11] and tetraquarks [12, 13]. The results are reviewed in Ref. [14].

In Sec. II we briefly discuss the basic features of our approach. In Sec. III we give more details on the compositeness condition, the infrared confinement and calculational techniques. The calculation of the matrix elements of the decays $B_s \rightarrow J/\psi + \eta$ and $B_s \rightarrow J/\psi + \eta'$ is performed in

Sec. IV. In Sec. V the calculated results are reported and comparison with available experimental data is given. Finally, in Sec. VI we summarize our findings.

II. COVARIANT QUARK MODEL

The interaction Lagrangian (density) of the model

$$\mathcal{L}_{\text{int}} = g_H H(x) \cdot J_H(x) + \text{H.c.} \quad (1)$$

is constructed from the hadron field $H(x)$ and the quark current. The latter is in case of mesons written as

$$J_M(x) = \int dx_1 \int dx_2 F_M(x; x_1, x_2) \times \bar{q}_2^a(x_2) \Gamma_M q_1^a(x_1) \quad (2)$$

and makes appear that, within the model, the interaction is mediated only by the quarks and the gluons are absent. The form of the vertex function F_M is chosen such as to reflect the intuitive expectations about relative quark-hadron positions

$$F_M(x; x_1, x_2) = \delta^{(4)}(x - w_1 x_1 - w_2 x_2) \times \Phi_M \left[(x_1 - x_2)^2 \right], \quad (3)$$

where we require $w_1 + w_2 = 1$. We actually adopt the most natural choice

$$w_i = \frac{m_i}{m_1 + m_2}, \quad i = 1, 2 \quad (4)$$

where the barycenter of the hadron is identified with the barycenter of the quark system. The interaction strength $\Phi_M \left[(x_1 - x_2)^2 \right]$ is assumed to have a Gaussian form which is in the momentum representation written as

$$\tilde{\Phi}_M(-p^2) = \exp \left(\frac{p^2}{\Lambda_M^2} \right). \quad (5)$$

Here Λ_M is a hadron-related size parameter which is regarded as a free parameter of the model. Additional free parameters are the quark masses

$$\begin{aligned} m_{u,d} &= 0.235 \text{ GeV}, \quad m_s = 0.424 \text{ GeV}, \\ m_c &= 2.16 \text{ GeV}, \quad m_b = 5.09 \text{ GeV} \end{aligned} \quad (6)$$

and a universal cut-off parameter $\lambda_{\text{cut-off}}$, which is commented on in more details later in this text. The numerical values of these parameters as well as of size parameters for several hadrons were fixed by fitting the model to measurement data of simple quantities, such as leptonic decay

constants and electromagnetic decay widths [9]. The value of the size parameter for B_s meson is $\Lambda_{B_s} = 1.95$ GeV. The model thus has in total $N_H + 5$ parameters: for each of N_H hadrons one Λ parameter, four quark masses and one universal cut-off. The coupling constants g_M can be related to these parameters using the so-called *compositeness condition*, which is discussed in the following section.

III. COMPOSITENESS CONDITION, CALCULATION METHODS AND INFRARED CONFINEMENT

In this section we give more details about some issues related to the model. As announced previously, one of them is the compositeness condition.

The quark fields as well as the hadron field enter the interaction Lagrangian of the model (1) as elementary although, in nature, the hadrons are compound of quarks. An effort was made in theoretical physics to find an appropriate description of composite particles. The authors of the references [15, 16] argue, that the hadron fields renormalization constant $Z_H^{\frac{1}{2}}$ can be interpreted as the matrix element between the physical state and the corresponding bare state. The case $Z_H = 0$ thus corresponds to a state not containing the bare state and can be therefore properly interpreted as a bound state. This idea was used and introduced into the covariant quark model [17, 18]. The renormalization constant is expressed through the derivative of the meson mass operator and takes the form

$$Z_M = 1 - \frac{3g_M^2}{4\pi^2} \tilde{\Pi}'_M(m_M^2) = 0. \quad (7)$$

The coupling constants are in this way eliminated as free parameters. This not only gives the model more predictive power but also helps to stabilize model predictions over wide spectra of hadron data. The first step in calculation of physical observables is thus the determination of couplings g_M of participating hadrons.

In the case of the pseudoscalar and vector mesons the derivative of the meson mass operator can be calculated in the following way

$$\begin{aligned} \tilde{\Pi}'_P(p^2) &= \frac{1}{2p^2} p^\alpha \frac{d}{dp^\alpha} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P^2(-k^2) \times \text{tr} \left[\gamma^5 S_1(k + w_1 p) \gamma^5 S_2(k - w_2 p) \right] \\ &= \frac{1}{2p^2} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P^2(-k^2) \times \left\{ w_1 \text{tr} \left[\gamma^5 S_1(k + w_1 p) \not{p} S_1(k + w_1 p) \gamma^5 S_2(k - w_2 p) \right] \right. \\ &\quad \left. - w_2 \text{tr} \left[\gamma^5 S_1(k + w_1 p) \gamma^5 S_2(k - w_2 p) \not{p} S_2(k - w_2 p) \right] \right\}, \end{aligned}$$

$$\begin{aligned}
\tilde{\Pi}'_V(p^2) &= \frac{1}{3} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \times \frac{1}{2p^2} p^\alpha \frac{d}{dp^\alpha} \int \frac{d^4 k}{4\pi^2 i} \tilde{\Phi}_V^2(-k^2) \times \text{tr} \left[\gamma^\mu S_1(k + w_1 p) \gamma^\nu S_2(k - w_2 p) \right] \\
&= \frac{1}{3} \left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{1}{2p^2} \int \frac{d^4 k}{4\pi^2 i} \times \tilde{\Phi}_V^2(-k^2) \\
&\quad \times \left\{ w_1 \text{tr} \left[\gamma^\mu S_1(k + w_1 p) \not{p} S_1(k + w_1 p) \gamma^\nu S_2(k - w_2 p) \right] \right. \\
&\quad \left. - w_2 \text{tr} \left[\gamma^\mu S_1(k + w_1 p) \gamma^\nu S_2(k - w_2 p) \not{p} S_2(k - w_2 p) \right] \right\}, \tag{8}
\end{aligned}$$

where $\tilde{\Phi}_M(-k^2)$ is the Fourier transform of the vertex function $\Phi_M[(x_1 - x_2)^2]$ and $S_i(k)$ is the quark propagator. We have used free fermion propagators for the quarks given by

$$S_q(k) = \frac{1}{m_q - \not{k}} \tag{9}$$

with an effective constituent quark mass m_q .

In computation of Feynman diagrams we use, in the momentum space, the Schwinger representation of the quark propagator

$$\begin{aligned}
S_q(k) &= \frac{m_q + \not{k}}{m_q^2 - k^2} \\
&= (m_q + \not{k}) \int_0^\infty d\alpha e^{-\alpha(m^2 - k^2)} \tag{10}
\end{aligned}$$

The general form of a resulting Feynman diagrams is

$$\Pi(p_1, \dots, p_m) = \int_0^\infty d^n \alpha \int [d^4 k]^\ell \Phi \times \exp \left\{ - \sum_{i=1}^n \alpha_i [m_i^2 - (K_i + P_i)^2] \right\}, \tag{11}$$

where K_i represents a linear combination of loop momenta, P_i stands for a linear combination of external momenta and Φ refers to the numerator product of propagators and vertex functions. The evaluation of these expressions can be much simplified if done in a smart way. One can use two operator identities, of which the first one

$$\begin{aligned}
\int d^4 k P(k) e^{2kr} &= \int d^4 k P \left(\frac{1}{2} \frac{\partial}{\partial r} \right) e^{2kr} \\
&= P \left(\frac{1}{2} \frac{\partial}{\partial r} \right) \int d^4 k e^{2kr} \tag{12}
\end{aligned}$$

is suited for an elegant loop momenta integration. The second one

$$\int_0^\infty d^n \alpha P \left(\frac{1}{2} \frac{\partial}{\partial r} \right) e^{-\frac{r^2}{\alpha}} = \int_0^\infty d^n \alpha e^{-\frac{r^2}{\alpha}} P \left(\frac{1}{2} \frac{\partial}{\partial r} - \frac{r}{\alpha} \right) \mathbf{1}, \tag{13}$$

where $r = r(\alpha_i)$ and $a = a(\Lambda_M, \alpha_i)$, simplifies the computation following the trace evaluation: the polynomial in the derivative operator which results from the trace can be applied to an identity, instead being applied to a more complicated exponential function.

The last point which remains to be discussed is the cut-off we apply in the integration over the Schwinger parameters. This integration is multidimensional with the limits from 0 to $+\infty$. In order to arrive to a single cut-off parameter we firstly transform the integral over an infinite space into an integral over a simplex convoluted with only one-dimensional improper integral. For that purpose we use the δ -function form of the identity

$$\mathbb{1} = \int_0^\infty dt \delta\left(t - \sum_{i=1}^n \alpha_i\right) \quad (14)$$

from which follows

$$\Pi = \int_0^\infty dt t^{n-1} \int_0^1 d^n \alpha \delta\left(t - \sum_{i=1}^n \alpha_i\right) \times W(t\alpha_1, \dots, t\alpha_n), \quad (15)$$

where W represents the integrand of Schwinger parameters. The cut-off λ is then introduced in a natural way

$$\int_0^\infty dt t^{n-1} \dots \rightarrow \int_0^{1/\lambda^2} dt t^{n-1} \dots \quad (16)$$

Such a cut-off makes the integral to be an analytic function without any singularities. In this way all potential thresholds in the quark loop diagrams are removed together with corresponding branch points [8]. Within covariant quark model the cut-off parameter is universal for all processes and its value, as obtained from a fit to data, is

$$\lambda_{\text{cut-off}} = 0.181 \text{ GeV}. \quad (17)$$

The integrals are computed numerically.

IV. DECAYS $B_s \rightarrow J/\psi + \eta$ AND $B_s \rightarrow J/\psi + \eta'$

The diagram of the $B_s \rightarrow J/\psi + \eta^{(\prime)}$ decay is shown in Fig. 1a. The η and η' particles are mixture of the light and the s -quark component. In the approximation $m_u = m_d \equiv m_q$ one can write their quark content as

$$\begin{aligned} \eta &: -\frac{1}{\sqrt{2}} \sin \delta (u\bar{u} + d\bar{d}) - \cos \delta (s\bar{s}) \\ &= -\sin \delta (q\bar{q}) - \cos \delta (s\bar{s}), \end{aligned} \quad (18)$$

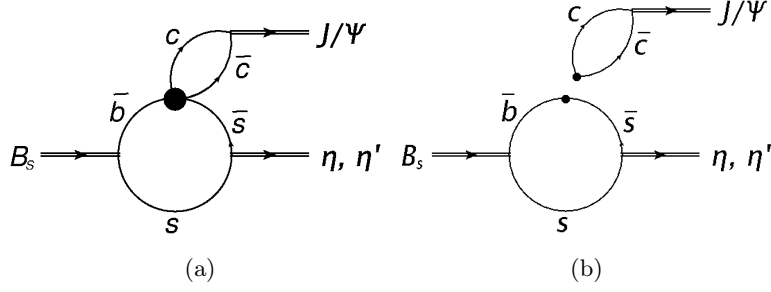


Figure 1: a) Diagram of $B_s \rightarrow J/\psi + \eta^{(\prime)}$ decay. b) Factorization of the diagram.

$$\begin{aligned} \eta' &: \frac{1}{\sqrt{2}} \cos \delta (u\bar{u} + d\bar{d}) - \sin \delta (s\bar{s}) \\ &= \cos \delta (q\bar{q}) - \sin \delta (s\bar{s}), \end{aligned} \quad (19)$$

with

$$q\bar{q} = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \quad (20)$$

and

$$\begin{aligned} \delta &= \theta_P - \theta_I, \\ \theta_I &= \arctan \frac{1}{\sqrt{2}} \approx 35.26^\circ, \\ \theta_P &\approx -13.34^\circ. \end{aligned} \quad (21)$$

The value of the pseudoscalar angle θ_P is deduced from the Ref. [19], where however a different convention is used for the mixing angle φ_P . It can be related to ours by $\delta = \varphi_P - \pi/2$.

We describe the decay only through the dominant s -quark contribution to the $\eta^{(\prime)}$ meson since the light quark one results from higher-order diagrams. The quark-hadron interaction is described by the Lagrangians from the Section II (see Appendix A 1) and we use an effective theory with four-quark interaction to describe the production of the J/ψ particle. The Lagrangian of this interaction is written as

$$\mathcal{L}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \sum_i C_i Q_i, \quad (22)$$

where G_F is the Fermi Coupling Constant, V_{xy} refers to the elements of the CKM matrix, C_i are the Wilson coefficients (Ref. [20]) and Q_i are operators listed in Appendix A 2. To each component of both η and η' particles we associate a size parameter. So, in general, the description requires four free parameters $\Lambda_{\eta}^{q\bar{q}}$, $\Lambda_{\eta}^{s\bar{s}}$, $\Lambda_{\eta'}^{q\bar{q}}$ and $\Lambda_{\eta'}^{s\bar{s}}$. We consider the mixing angle δ as fixed although,

in principle, it also might be varied to determine its most suited value within the covariant quark model. As a model-independent parameter it can be compared to other models and to data.

An important simplification in calculations comes from the evaluation of the Q_i operator matrix elements. It can be shown that the expression factorizes into a part that corresponds to the $B_s \rightarrow \eta^{(\prime)}$ transition form factor and a part that is proportional to the decay constant of the J/ψ particle (Fig 1b and Appendix A 3).

It is readily seen from the Fig. 1b that only the strange quark component gives the contribution to the color-suppressed $B_s \rightarrow \eta^{(\prime)}$ decays. The invariant matrix element describing this decay is written as

$$M[B_s(p_1) \rightarrow P_{ss}(p_2) + J/\psi(q)] = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^\dagger C_W F_+^{B_s P_{ss}}(q^2) m_{J/\psi} f_{J/\psi} (p_1 + p_2) \cdot \epsilon_{J/\psi}, \quad (23)$$

where $q = p_1 - p_2$, $q \cdot \epsilon_{J/\psi} = 0$, $q^2 = m_{J/\psi}^2$ and the color factor $C_W = C_1 + \xi C_2 + C_3 + \xi C_4 + C_5 + \xi C_6$ with $\xi = 1/N_c$. The terms multiplied by the color factor ξ will be dropped in the numerical calculations according to the $1/N_c$ expansion.

The leptonic decay constants of the pseudoscalar and vector mesons are defined by

$$\begin{aligned} N_c g_P \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_P(-k^2) \text{tr} \left[O^\mu S_1(k + w_1 p) \gamma^5 S_2(k - w_2 p) \right] &= f_P p^\mu, \quad p^2 = m_P^2, \\ N_c g_V \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_V(-k^2) \text{tr} \left[O^\mu S_1(k + w_1 p) \not{\epsilon}_V S_2(k - w_2 p) \right] &= m_V f_V \epsilon_V^\mu, \quad p^2 = m_V^2. \end{aligned} \quad (24)$$

The form factors of the $P_{[\bar{q}_3 q_2]} - P'_{[\bar{q}_1 q_3]}$ transition are given by

$$\begin{aligned} \langle P'_{[\bar{q}_1 q_3]}(p_2) | \bar{q}_2 O^\mu q_1 | P_{[\bar{q}_3 q_2]}(p_1) \rangle &= N_c g_P g_{P'} \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}_P \left(- (k + w_{13} p_1)^2 \right) \tilde{\Phi}_{P'} \left(- (k + w_{23} p_2)^2 \right) \\ &\quad \times \text{tr} \left[O^\mu S_1(k + p_1) \gamma^5 S_3(k) \gamma^5 S_2(k + p_2) \right] \\ &= F_+(q^2) P^\mu + F_-(q^2) q^\mu \end{aligned} \quad (25)$$

where in addition to Eq. (4) we have introduced a two-subscript notation $w_{ij} = m_j/(m_i + m_j)$ such that $w_{ij} + w_{ji} = 1$. The calculation of the widths is straightforward. One has

$$\begin{aligned} \Gamma(B_s \rightarrow J/\psi + \eta) &= \frac{G_F^2}{4\pi} |V_{cb} V_{cs}^\dagger|^2 C_W^2 f_{J/\psi}^2 |\mathbf{q}_\eta|^3 \cos^2 \delta \left(F_+^{B_s \eta}(m_{J/\psi}^2) \right)^2, \\ \Gamma(B_s \rightarrow J/\psi + \eta') &= \frac{G_F^2}{4\pi} |V_{cb} V_{cs}^\dagger|^2 C_W^2 f_{J/\psi}^2 |\mathbf{q}_{\eta'}|^3 \sin^2 \delta \left(F_+^{B_s \eta'}(m_{J/\psi}^2) \right)^2, \end{aligned} \quad (26)$$

where $|\mathbf{q}_P| = \lambda^{1/2}(m_{B_s}^2, m_P^2, m_{J/\psi}^2)/(2m_{B_s})$ is the momentum of the outgoing particles in the rest frame of the decaying particle.

Observable	Covariant	Experiment		Other models		
	quark model			Münz [21]	Becchi [22]	Jaus [23]
$\Gamma_{\eta \rightarrow \gamma\gamma}$	0.380 keV	0.511 ± 0.028 keV	[24]	0.440 keV	-	0.485 keV
$\Gamma_{\eta' \rightarrow \gamma\gamma}$	3.74 keV	4.34 ± 0.14 keV	[25]	2.90 keV	-	2.90 keV
$\Gamma_{\rho^0 \rightarrow \eta\gamma}$	53.07 keV	44.73 ± 2.99 keV	[25]	-	44 keV	59 keV
$\Gamma_{\omega \rightarrow \eta\gamma}$	6.21 keV	3.91 ± 0.34 keV	[25]	-	6.4 keV	8.7 keV
$\Gamma_{\eta' \rightarrow \omega\gamma}$	9.49 keV	5.47 ± 0.63 keV	[25]	-	-	4.8 keV
$\Gamma_{\varphi \rightarrow \eta\gamma}$	42.59 keV	58.90 ± 2.45 keV	[25]	-	304 keV	55.3 keV
$\Gamma_{\varphi \rightarrow \eta'\gamma}$	0.276 keV	0.281 ± 0.015 keV	[25]	-	-	0.57 keV
$\mathcal{B}_{B_d \rightarrow J/\psi + \eta}$	16.5×10^{-6}	$(12.3 \pm 1.9) \times 10^{-6}$	[26]	-	-	-
$\mathcal{B}_{B_d \rightarrow J/\psi + \eta'}$	12.2×10^{-6}	$< 7.4 \times 10^{-6}$	[26]	-	-	-
$\mathcal{B}_{B_s \rightarrow J/\psi + \eta}$	4.67×10^{-4}	$(5.10 \pm 1.12) \times 10^{-4}$	[1]	-	-	-
$\mathcal{B}_{B_s \rightarrow J/\psi + \eta'}$	4.04×10^{-4}	$(3.71 \pm 0.95) \times 10^{-4}$	[1]	-	-	-

Table I: Decay widths and branching fractions for selected processes: model predictions and data.

In addition to the Belle results, we need further data in order to over-constrain the model. We have chosen the following decays

$$\begin{aligned}
&\eta \rightarrow \gamma\gamma & \varphi \rightarrow \eta\gamma \\
&\eta' \rightarrow \gamma\gamma & \varphi \rightarrow \eta'\gamma \\
&\rho^0 \rightarrow \eta\gamma & B_d \rightarrow J/\psi + \eta \\
&\omega \rightarrow \eta\gamma & B_d \rightarrow J/\psi + \eta' \\
&\eta' \rightarrow \omega\gamma &
\end{aligned} \tag{27}$$

These processes have already been previously described by the covariant quark model [8] and can be straightforwardly included into a global fit.

V. RESULTS AND DISCUSSION

The optimal model parameters (in GeV)

$$\begin{aligned}
\Lambda_{\eta}^{q\bar{q}} &= 0.881, & \Lambda_{\eta}^{s\bar{s}} &= 1.973 \\
\Lambda_{\eta'}^{q\bar{q}} &= 0.257, & \Lambda_{\eta'}^{s\bar{s}} &= 2.797
\end{aligned} \tag{28}$$

were obtained from a χ^2 fit to the data. The corresponding description of the data by the covariant quark model is shown in Table I.

First, we would like to discuss the ratio of the branching fractions of $B_s \rightarrow J/\psi + \eta'$ and $B_s \rightarrow J/\psi + \eta$ decays which has been measured by both, Belle [1] and LHCb [2] collaborations. They reported the following values for this ratio

$$R \equiv \frac{\Gamma(B_s \rightarrow J/\psi + \eta')}{\Gamma(B_s \rightarrow J/\psi + \eta)} = \begin{cases} 0.73 \pm 0.14 \pm 0.02 & \text{Belle [1]} \\ 0.90 \pm 0.09^{+0.06}_{-0.02} & \text{LHCb [2]} \end{cases} \quad (29)$$

As follows from the Eq. 26 the ratio R is defined by

$$R^{\text{theor}} = \underbrace{\frac{|\mathbf{q}_{\eta'}|^3}{|\mathbf{q}_{\eta}|^3}}_{\approx 1.04} \tan^2 \delta \times \underbrace{\left(\frac{F_+^{B_s \eta'}}{F_+^{B_s \eta}} \right)^2}_{\approx 0.83} \approx 0.86. \quad (30)$$

One can see that the nontrivial dependence of the form factor F_+ on the η and η' masses and size parameters provides a reduction of the model independent result 1.04 to 0.86.

The interpretation of the results might be done in several steps. Firstly, one observes that all model predictions are of the order of the experimental numbers. Majority of them have the relative error smaller than 30%, when compared to the data. The most important relative error is 73% in case of $\Gamma_{\eta' \rightarrow \omega \gamma}$, still significantly smaller than factor two. The latter case actually suggests one might want to consider the possibility of a gluonium content of the $\eta^{(\prime)}$ meson, as discussed in the Ref. [19] (and other references therein). On the other hand one must admit that, when the experimental errors are taken into account, the model predictions are usually quite outside the error intervals. Here one can argue in two ways. Firstly, the model is only an approximation to the first-principle theory. One thus, from beginning, does not expect the model to be fully accurate. Consequently, the goodness of the model when interpreted through the data error intervals depends on the data precision measurement. Every approximate model becomes “very bad” when the data becomes very precise. So the point of view based on the error intervals might not be the most suited one.

An optional criterion might be a comparison to the experimental needs. Experiments usually need a model that allows for an appropriate correction of detector effects. In fact they usually need more models so as to be able to establish a systematic error related to the data correction. We think that within this logic, the results of the covariant quark model make the model fully acceptable and legitimate is development and eventual use.

Yet, an additional approach to rate the model is to compare it to other existing models [21–23, 27]. We have chosen some of not very numerous works, that give explicit numbers for a set of observables overlapping with those chosen by us. When comparing processes in common (Table I),

none of these models describes the data better than ours (if total the χ^2 is calculated). This fact confirms, that the covariant quark model is a very competitive one among the available models.

Finally, one can still await further confirmation and more precision in the experimental data, especially in the case of recent results obtained by a single collaboration which have not yet been independently cross-checked. The final picture concerning the data description by the model might still change.

VI. SUMMARY

We have calculated the matrix elements and branching fractions of the decays of B_s into $J/\psi + \eta'$ and $J/\psi + \eta$ in the framework of the covariant quark model. We have used the model parameters which have been fixed in our previous papers except the size parameters characterizing the distributions of non-strange and strange quarks within the η and η' . We fix them by fitting the available experimental data on two-body electromagnetic decays involving the η and η' and the above B_s decay. In particular, we have found that the ratio of the branching fractions of the decays B_s into $J/\psi + \eta'$ and $J/\psi + \eta$ is equal to $R \approx 0.86$ in agreement with the data reported by Belle and LHCb collaborations.

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Appendix A: Expressions and Formulas

We make use of this Appendix to display longer formulas referred in the text.

1. Lagrangian of the model

The Lagrangian (density) is written as

$$\mathcal{L} = \mathcal{L}_{B_s} + \mathcal{L}_\eta + \mathcal{L}_{J/\psi} + \mathcal{L}_{\text{eff}}$$

with \mathcal{L}_{eff} being given in the text.

$$\mathcal{L}_{B_S}(x) = g_{B_S} \bar{B}_S^0(x) \iint dx_1 dx_2 \delta(x - w_b x_1 - w_s x_2) \phi_{B_S}[(x_1 - x_2)^2] \bar{b}(x_1) i\gamma^5 s(x_2) + \text{h. c.}$$

$$\begin{aligned} \mathcal{L}_\eta(x) = g_\eta \eta(x) \iint dx_1 dx_2 \delta\left(x - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) \phi_\eta[(x_1 - x_2)^2] \\ \times \left\{ -\frac{1}{\sqrt{2}} \sin(\delta) [\bar{u}(x_1) i\gamma^5 u(x_2) + \bar{d}(x_1) i\gamma^5 d(x_2)] - \cos(\delta) [\bar{s}(x_1) i\gamma^5 s(x_2)] \right\} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\eta'}(x) = g_{\eta'} \eta'(x) \iint dx_1 dx_2 \delta\left(x - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) \phi_{\eta'}[(x_1 - x_2)^2] \\ \times \left\{ \frac{1}{\sqrt{2}} \cos(\delta) [\bar{u}(x_1) i\gamma^5 u(x_2) + \bar{d}(x_1) i\gamma^5 d(x_2)] - \sin(\delta) [\bar{s}(x_1) i\gamma^5 s(x_2)] \right\} \end{aligned}$$

$$\mathcal{L}_{J/\psi} = g_\psi \psi_\mu(x) \iint dx_1 dx_2 \delta\left(x - \frac{1}{2}x_1 - \frac{1}{2}x_2\right) \phi_\psi[(x_1 - x_2)^2] \bar{c}(x_1) \gamma^\mu c(x_2)$$

2. Four-quark vertex operators

The four-quark operators read as follows

$$\begin{aligned} Q_1 &= (\bar{c}_{a_1} b_{a_2})_{V-A} (\bar{s}_{a_2} c_{a_1})_{V-A} & Q_4 &= (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V-A} \\ Q_2 &= (\bar{c}_{a_1} b_{a_1})_{V-A} (\bar{s}_{a_2} c_{a_2})_{V-A} & Q_5 &= (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V+A} \\ Q_3 &= (\bar{s}_{a_1} b_{a_1})_{V-A} (\bar{c}_{a_2} c_{a_2})_{V-A} & Q_6 &= (\bar{s}_{a_1} b_{a_2})_{V-A} (\bar{c}_{a_2} c_{a_1})_{V+A} \end{aligned}$$

with

$$(\bar{\psi}\psi)_{V-A} = \bar{\psi} O^\mu \psi, \quad O^\mu = \gamma^\mu (1 - \gamma^5) \quad (\bar{\psi}\psi)_{V+A} = \bar{\psi} O_+^\mu \psi, \quad O_+^\mu = \gamma^\mu (1 + \gamma^5).$$

3. Some elements on factorization

For the η particle and the operator Q_1 , the time-product matrix element is written as

$$\left\langle T \left\{ [\bar{b}(x_1^B) i\gamma^5 s(x_2^B)] C_1 Q_1(x^w) [\bar{s}(x_1^\eta) i\gamma^5 s(x_2^\eta)] [\bar{c}(x_1^\psi) \hat{e}_\psi c(x_2^\psi)] \right\} \right\rangle_0.$$

This, after evaluating the contractions and color indices, leads to

$$\begin{aligned} \langle T \{ \dots Q_1 \dots \} \rangle_0 &= -C_1 N_c^2 \epsilon_{\psi, \nu} \times \text{Tr} [S_b(x^w - x_1^B) \gamma^5 S_s(x_2^B - x_1^\eta) \gamma^5 S_s(x_2^\eta - x^w) O^\mu] \\ &\times \text{Tr} [S_c(x^w - x_1^\psi) \gamma^\nu S_c(x_2^\psi - x^w) O^\mu], \end{aligned}$$

where S is a propagator, index w refers to the position of the four-quark interaction and $\epsilon_{\psi,\nu}$ is the J/ψ polarization vector. The structure of the expression makes visible the factorization into a “ $B_s - \eta$ ” and “ J/ψ ” part. When actually comparing the two parts to the expressions from [9], one recognizes the form factor and the decay constant. Situation is analogical for η' and other operators.

Further, very simple relations exist between the matrix elements for different operators. One has

$$\langle T \{ \dots Q_{2,4,6} \dots \} \rangle_0 = \frac{1}{N_c} \langle T \{ \dots Q_{1,3,5} \dots \} \rangle_0.$$

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